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Modeling surface structure of tribotechnical materials

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Abstract

Modern tribology makes it possible to correctly calculate, diagnose, predict and select appropriate materials for friction pairs, to determine the optimal mode of operation of the tribo-joint. The main parameter for solving friction problems and other problems of tribology is the topography of the surface. The main purpose of the models in these tasks is to display the tribological properties of engineering surfaces. In the framework of the classical approach, the topography of the surface is studied on the basis of its images from the point of view of functional and statistical characteristics: the evaluation of the functional characteristics is based on the maximum roughness along the height and the average roughness along the center line, and the statistical characteristics are estimated using the power spectrum or the autocorrelation function. However, these characteristics are not only surface properties. They depend on the resolution of the device for measuring the surface geometry and the length of the scan. However, the degree of complexity of a surface shape can be represented by a parameter called the fractal dimension: a higher degree of complexity has a larger value of this parameter. Fractal dimensionality is a characteristic of surface relief and makes it possible to explain tribological phenomena without the influence of resolution. This article provides an overview of mathematical approaches to the description of the relief of engineering surfaces, in particular statistical, stochastic and topological modeling, their limitations, advantages and disadvantages. The implementation of the principles of the theory of fractal structures is discussed, which makes it possible to introduce the degree of imbalance of the tribological system into the analysis of structure formation in the surface and near-surface layers of materials and to describe the development of friction and wear processes. This is the basis for controlling the structure of the surface layers of materials with given properties. The concept of fractals, used for the quantitative description of the dissipative structure of the tribojunction zone, makes it possible to establish a connection between its fractal dimension and mechanical properties, as well as critical states of deformation of metals and alloys. The course of research and stages of fractal modeling, the classification of methods of fractal analysis of the structure of engineering contact surfaces are considered. A critical analysis of modern models based on the energy-spectral density function, which are quite similar to fractal models, is presented. Readers are expected to gain an overview of research developments in existing modeling methods and directions for future research in the field of tribology.

Keywords:surface relief; statistical models; stochastic models; fractal models; energy spectral density function.

Introduction

Tribotechnical indicators of materials (compatibility, wear resistance, antifriction, etc.) characterize the behavior of the entire tribological system as a whole [1]. Therefore, it is not possible to establish a connection between the above indicators and the geometric and/or physical-mechanical-chemical properties of the elements of the friction pair.

The article deals mainly with engineering surfaces, that is, those used in engineering practice. It is known that all technical surfaces are rough [1, 2, 3], so contact between technical surfaces is carried out using several contact points [4]. If the surface profile $z(x)$ is determined using the Fourier distribution, and the term "roughness" is defined as a short-wave form, then the technical surface is defined as a long-wave form and is called a "wave-like" surface [5]. If the waviness is removed from the surface profile, the rough surface can be considered nominally flat [6]. The roughness of technical surfaces is a decisive factor for the performance of tribological components. The surface profile has a huge influence on energy dissipation during sliding of dry engineering surfaces and, accordingly, on friction [7].

Increasing the reliability of many technical systems is impossible without an in-depth study of the processes occurring on friction surfaces; development of physical ideas about friction and wear; application of modern research methods based on the results and methods used in classical fundamental and applied physical and mathematical sciences; use of computer technologies.

This article presents a critical review of some popular functional, statistical, fractal, and related methods for modeling and analyzing surface roughness. Prospective trends in the development of mathematical modeling in tribology are proposed, determined on the basis of the obtained data and statistics of the published literature in this field. After all, the choice of appropriate surface characterization methods and calculation methods for the study of various surfaces is the main problem of current studies of engineering surface topography.

Modern mathematical modeling in the study of the mechanism of contact and destruction of engineering friction surfaces develops in the following main directions: statistical modeling, stochastic modeling, topological modeling, fractal modeling.

Probabilistic and statistical characteristics of surface roughness

Modern research involves a systematic approach to the study of tribotechnical problems. The importance of such an approach increases in the case of applying probabilistic statistical methods in solving problems in the field of friction, since this process is quite complex and has a stochastic nature of functioning. The method of creating mathematical models of the friction and wear process using the apparatus of the theory of similarity, dimensions and mathematical planning of the experiment is quite progressive, since the transition to generalized coordinates sharply reduces the number of factors that must be taken into account and gives sufficiently justified values of the initial parameters. Most tribological systems work in accordance with the Pareto principle, which states that only some of the many factors are significant from the point of view of the system's characteristics. The methods of group consideration of arguments, a priori ranking of factors, rank correlation, random balance and others are used to determine essential factors. The rational choice of the appropriate method is determined by the availability of a priori information about the researched object. Regression analysis is widely used to establish the relationship between input and output parameters and to obtain a mathematical model adequate for the object under study.

One of the first attempts to apply statistical methods to describe surface roughness was presented by Abbott and Firestone, who calculated the cumulative distribution function of surface heights:

$$\Phi(z) = \int_z^{\infty} \varphi(t)\mathrm{dt},$$

where $\phi(z)$ is the probability density function.

In tribology, this parameter is called the Abbott-Firestone curve or the bearing area curve. Subsequently, a huge number of statistical roughness parameters were introduced [8]. These characteristics were related to both the vertical distribution of heights and the horizontal distribution of rough profiles [9].

The next step in surface roughness research was the idea of modeling based on the theory of random processes. This idea was first implemented by Linnik and Khusu [10], who suggested using the details of the stationary Gaussian random process graph and the correlation function for the Gaussian random process N(x) to describe the surface roughness:

$$N(x) = N(0) \cdot e^{(-\alpha|x|)}, \tag{1}$$

where N(0) and α are some roughness parameters. A similar idea was presented later by Whitehouse and Archard [11]. They proposed to describe the Gaussian profile z(x) of a random rough surface by the distribution of the heights of its protrusions and the correlation (autocorrelation) function of the process R(δ):

$$R(\delta) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [z(x+\delta) - \bar{z}]\,[z(x) - \bar{z}]\mathrm{dx}, \tag{2}$$

where $\bar{z}$ is the middle line of the profile.

Statistical modeling results are effective if the roughness is Gaussian (normal). If the roughness is not normal, then the properties of the sample trajectory are not fully determined by the mean and covariance functions. Therefore, the statistical modeling technique includes a stage of assessing the reliability of the model, which is based on proving the assumption of a Gaussian (normal) distribution of the heights of the projections of the rough surface.

There are many criteria for testing this assumption. Each of these criteria provides a quantitative assessment of the closeness between a theoretical Gaussian distribution and an observed sample of measurements by calculating a p-value. Estimates are based on certain statistics of the relevant criterion. According to the literature review, the most popular criteria for checking the normality of roughness of different surfaces are: Pearson, Kolmogorov-Smirnov (KS), Anderson-Darling (AD), Cramer-von Mises (CVM), Shapiro-Wilk (SW), Shapiro-Francia (SF) criteria), Lilliefors (LF) [12]. The p-value is a number that characterizes, for the observed measurements, the significance on a scale of [0, 1] that the hypothesis of a normal distribution law is true. As a rule, an acceptable level of significance is nominated (5%). The trend of using statistical tests on both nano and micro scales is relevant.

Non-Gaussian processes can be generated by a stochastic differential equation:

$$dX(x) = -\theta(X(x) - \mu)dx + \sigma(X(x))dWB(x),$$

where $x \geq 0$ and $WB(x)$ is a standard Brownian motion (Wiener process). Choosing the appropriate value of the parameter μ and the function $\sigma(\cdot)$, we obtain a certain distribution of the process $X(x)$ by height with the autocorrelation function $\rho(x) = e{-}\theta|x|$, by the power spectrum $G(\omega)=2\pi\theta/(\theta 2+\omega 2)$ for any choice of μ and $\sigma(\cdot)$.

Research has proven that the surface relief is a non-stationary random process, that is, this statistical parameter depends on the scale. In other words, the accuracy of this characteristic parameter of the contact problem is affected by the length of the sample and the resolution of the measuring device [13].

Statistical models of contact with multiple protrusions between two nominally flat surfaces are the most popular for predicting the contact behavior of rough surfaces, their assumptions and simplifications greatly limit their reliability, and the criteria for identifying protrusions and their characteristics lead to significant deviations in the calculated topographic input parameters, which are also strongly dependent from the resolution of the topography measurement technique. Typical engineering surfaces are also not isotropic, and the distribution of ledge heights is not Gaussian [14].

Methods of topological modeling of the structure of the surface layer

Traditional methods of topological modeling (geometric assessment) of the formation of various objects, including in tribology, are based on the approximate approximation of the structure of the object under study (in tribology of surface and near-surface layers) by geometric shapes, for example, lines, segments, planes, polygons, polyhedra, spheres. These techniques are based on classical Euclidean geometry, the topological dimension of which is an integer. At the same time, the internal structure of the object under study is usually ignored, and the processes of structure formation and their interaction with each other and with the environment are characterized by integral thermodynamic parameters. This, naturally, leads to the loss of a significant part of information about the properties and behavior of the studied systems, which, in fact, are replaced by more or less adequate models. In some cases, such a replacement is quite justified. However, there are problems when the use of topologically non-equivalent models is fundamentally unacceptable. In particular, for the modeling of structurally complex objects, where a generalized concept of a specific physical representation of the structure and description of the properties of the object is necessary.

An example of topological modeling of elastic contact between two nominally flat metal surfaces is the Greenwood-Williamson model [6]. The surface relief model is a set of spherical segments having the same radius of rounding of the upper part of the protrusions and located on the middle plane of the rough surface. The model is based on fairly clear physical provisions about the contact interaction of rough surfaces in the elastic state of frictional contact spots (the number of contacting spheres of a certain height increases when the surfaces approach each other) [6]. Adopting a constant radius of the upper part of the protrusions simplifies the modeling of the contact interaction process, while the accuracy of the calculations decreases. And at low loads, when we have to take into account sub-roughness when determining the contact parameters (when the contact between two rough surfaces consists of a large number of contact spots of different sizes), the Greenwood-Williamson model is inapplicable. Majumdar [14] managed to eliminate the shortcomings inherent in the Greenwood-Williamson model using fractal modeling.

Fractals Approaches to Surface Topography

In fact, the fractal terminology for describing surface roughness was pioneered by Berry and Hannay [16], who argued that the geometric properties of rough surfaces can be characterized by a new concept of "fractal", which was described in detail by Mandelbrot [17]. He introduced the concepts of fractal, fractal geometry and fractal dimension (FD).

Fractal geometry became widespread in tribology thanks to the works of A.-K. Janahmadov, V. Ivanova [18, 19], which are devoted to the analysis and control of structure formation in alloys, surface and near-surface layers of materials as open nonlinear systems that are far from a state of thermodynamic equilibrium. Such systems are unbalanced due to the dissipation of energy received from the outside. As a result of self-organization, stable structures can arise in such systems, which exist under the condition of constant dissipation, that is, loss of energy by the system. With the appearance of a complex ordered structure in the system, entropy increases, which is compensated by a negative flow of entropy from the outside.

To date, it has been established that the resistance to destruction of metals and alloys is determined by the dynamic structure that is formed in the process of deformation and has dissipative properties. In tribology, surface layers and all internal boundaries should be considered as an independent planar nonlinear subsystem with broken translational invariance, which is the leading functional subsystem in a deformed solid. The main part of the stresses arising during friction is concentrated in the near-surface layers of the friction elements. The reconstruction of the surface layer under the action of external thermal loads occurs precisely in the process of establishing the temperature field and is a process of dissipative structure formation associated with deformation defects [19].

Self-organized dissipative structures in open systems are fractal [17]. This makes it possible to apply fractal modeling when studying the physical and mechanical nature of the destruction of materials by introducing new quantitative indicators of structures in the form of fractal dimensions.

The basis of fractal modeling is the concept of a fractal - a self-similar structure with a fractional dimension, which has the property of scale invariance. In general, fractals are a powerful tool for understanding and designing materials with complex structures and properties [17]. It is based on the works of Russ JC [20], Mandelbrot [17], Feder [21], and others.

The fractal dimension FD characterizes any self-similar system: when the linear dimensions change by u times, the fractal value changes by uFD times. The fractal dimension is not related to the topology, but to the method of construction of the considered object [21].

For a fractal structure, the dimensionality or, usually, the fractional parameter FD, describes the preservation of statistical characteristics when scaling. Fractal dimensionality allows you to quantitatively describe microstructures and their constituent elements, to establish the actual area of collision of phases, the actual lengths of "rough" lines and surfaces, and to determine other structural parameters related to the properties of materials. The fractional metric dimension of such objects not only characterizes their geometric image, but also reflects the processes of their formation and evolution, as well as determines their dynamic properties. Fractals provide a compact way of describing objects and processes in strictly quantitative terms.

The fractal model assumes that the engineering surface is self-similar (a part of the surface reflects the entire object) and scaling (a part repeats its structural features at a different measurement scale). Thus, the fractal approach has the potential to predict the behavior of a surface phenomenon at a particular length scale based on observations at other length scales.

The self-similarity of structures is established on the basis of the analysis of certain geometric patterns and their measurements at different magnification scales. In order to establish the fractality of the structure, it is necessary [22]: 1) to check self-similarity; 2) determine the limits of self-similarity; 3) calculate the fractal dimension.

The fractal tribomodeling methodology is explained in Figure 1, where the main stages of the research and their results are defined.

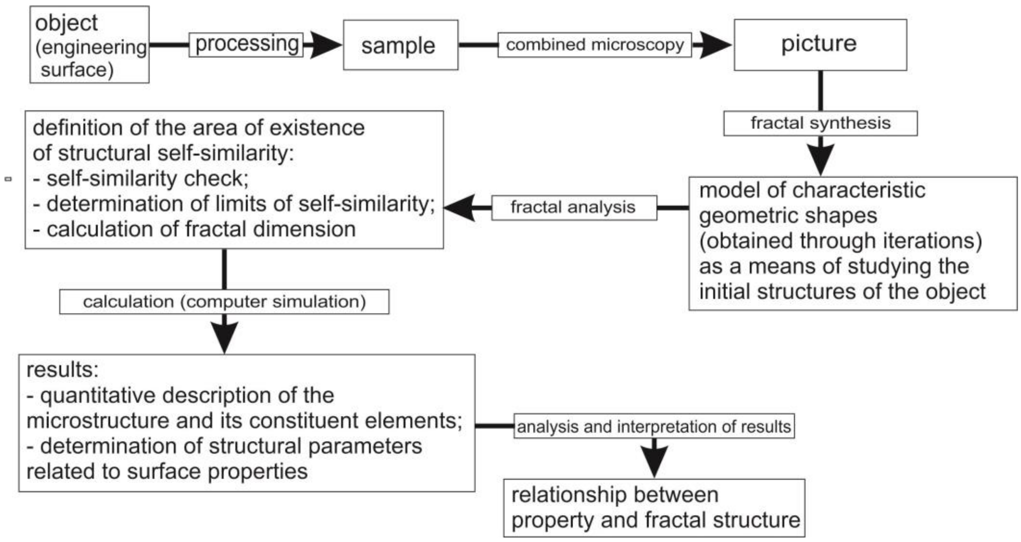

Fig. 1. Information model of the study of structural characteristics of the engineering surface based on fractal analysis

It should be noted that determining the relationship between a property and a fractal structure is a difficult task, since the existing models establishing these relationships for periodic structures are not applicable to fractal ones [23]. The solution of this problem requires the development of fractal analysis of microstructures, the determination of the area of existence of structural self-similarity, as well as the development of fractal synthesis, which includes the modeling of characteristic geometric shapes (through iterations) as a way to study initial structures in real materials.

Below (Figure 2) is a classification of the main experimental methods of studying statistically self-similar tribo-structures.

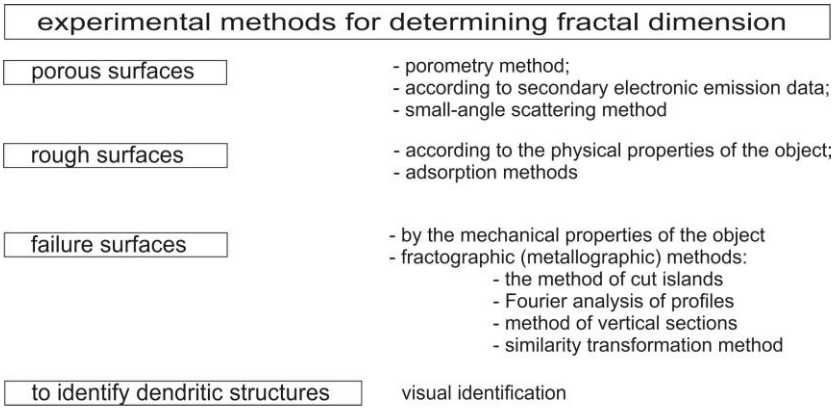

Fig. 2. Classification of experimental methods for determining the fractal dimension of statistically similar structures

It should be noted that fractographic (metallographic) studies are the most direct methods of determining the fractal dimension of statistically self-similar profiles and surfaces of natural objects.

Fractal theory is used as a mathematical model for random surface topography, which can be used as input in modeling contact mechanics. In many tribological applications, some geometric parameters defined in Euclidean space, such as the unfolded area, bearing surface, cavity, and material volume, are very difficult to measure independently of the unit of measurement. The values of these parameters increase when the measurement scale is reduced. Fractal geometry can be used as an adaptive space for rough morphology, in which the roughness can be considered as a continuous but non-differentiable function, and the FD dimension of this space is an intrinsic parameter to characterize the surface topography [15]. Fractal dimensionality is used as an indicator of the real values of various scale-dependent parameters, such as length, surface, and roughness volume, and as an invariant parameter for analyzing the distribution law of the area of contact points.

Real physical objects, which have signs of self-similarity, can rarely be described using only one value of the fractal dimension. That is why the analysis based on the theory of multifractals - non-homogeneous fractal objects - has become very popular recently. A characteristic of a multifractal is an infinite spectrum of such dimensions, which is called the generalized fractal dimension or Renyi dimension [21].

With the help of multifractal characteristics, phenomena in contact mechanics, wettability, and lubrication of rough material are described, where knowledge of the area of the supporting surface, the developed area, or the volume of voids is directly related to the scale of observation [22].

Another important step in advancing the fractal approach to the description of surface roughness was the study of the Weierstrass-Mandelbrot fractal function by Berry and Lewis [24]. Mandelbrot [17] generalized the Weierstrass function, the graph of which is continuous everywhere and nowhere differentiable, and introduced the complex-valued Weierstrass-Mandelbrot (WM) function W(x) and its special real case C(x; p):

$$W(x;p) = \sum_{n=-\infty}^{\infty} p^{-\beta n}\left(1 - e^{\mathrm{i}p^{n}x}\right)e^{i\varphi_n},$$
$$C(x;p) = \sum_{n=-\infty}^{\infty} p^{-\beta n}\left(1 - \cos(p^{n}x)\right),\ p > 1,\ 0<\beta<1. \quad (3)$$

where ϕn are arbitrary phases. Box-counting dimension (the Minkowski dimension) of graphs C(x; p) is equal to $D = 2 - \beta$. There is no rigorous mathematical proof that its Hausdorff dimension is the same. The plot of the function C(x; p) has often been used to model rough profiles.

Later, Weierstrass-type functions were used by many researchers as a model of rough surfaces [22].

For a while, fractal models were all too popular. There are reviews of the application of fractal concepts in contact problems, in fracture mechanics, and several articles on the use of fractal concepts in tribology [25, 26].

Thus, let's define some main features of the fractal approach.

1. The authors in [25] divided fractals into mathematical and physical (natural) fractals. Both mathematical and physical fractals use the concept of coverage. This means that the object (set) is covered by cubes of size greater than or equal to δ. Fractal geometry is based on mathematical fractals. Mathematical methods of fractal geometry are described in many books and articles where various FDs are studied as applied to mathematical objects. Various FDs are used in research, mainly Hausdorff dimension (dimH) and box-counting dimension (the Minkowski dimension) (dimB) (and the Hausdorff dimension of the set S may not be equal to the box-counting dimension of dimBS, but it is known that dimHS ≤ dimBS.) These FDs can be calculated by taking the limit at $\delta \to 0$ [17].

A mathematical fractal curve has an infinite length. Even if a mathematical fractal curve is continuous everywhere, it is non-differentiable. Therefore, it is often very difficult to formulate a boundary value problem for solids that have a fractal boundary.

If real-world objects or numerically modeled objects have a power-law number-radius relationship, then those objects are physical fractals. The power law of the number-radius ratio has the form:

$$N(\delta)\sim\delta-D,\ \delta^*\leq\delta\leq\ \delta^*,\ N(R)\sim(R/\delta)D,\ r^*\leq R\leq R^*, \tag{4}$$

where N(δ) is the number of elements covering the object of size δ, D is the dimension of the object FD, δ* and δ* are the upper and lower limits of the physical fractal law, respectively. The first relation (4) is used when the coverage size δ varies and the object size R is fixed, while the latter relation is used when the coverage size δ is fixed and the object size R varies. In the latter case, R* and r* are the upper and lower cutoff limits. The ratio ln(N(δ*)) / ln(R) is used to estimate the D value.

The main difference between these types of fractals is as follows: the physical behavior of the fractal (4) is observed only in a limited range of scales, while for the study of mathematical fractals it is necessary to take into account the scales of consideration up to the zero limit.

If FD is specified, it is convenient to use the fractional part FD – D*. Then FD of fractal profiles and surfaces are equal to 1 + D* and 2 + D*, respectively.

2. Self-similar sets are a very specific kind of fractals. In general, self-similarity is not related to mathematical fractals. Their scaling properties are based on the scaling of the fractal measure or quasi-measure [25], while for physical fractals their scaling properties are reflected by relations (4).

3. On average, the estimate of the FD value is 1.5. However, if the FD value is less than two or three orders of magnitude, the fractal concept is not useful [26].

4. In addition, the term fractal geometry is also quite often loosely applied to a set of semi-empirical or empirical methods for estimating the FD of objects. In general, the FD values obtained by different practical methods are not reliable [25].

5. As noted by Whitehouse [27], there is very little scatter in the FD values obtained for surfaces produced by different manufacturing processes. In addition, there is no well-established algorithm for estimating the intercepts of the fractal law (3).

6. The roughness of real bodies is not a mathematical fractal. In [25] using fractal parametrically homogeneous surfaces, it is shown that the tribological properties of a rough surface cannot be characterized only by the fractal dimension of the surface.

Fractals are only mathematical idealizations of complex forms of natural objects. Of course, it is possible to use a mathematical fractal as a possible model that reflects the power dependence of the number-radius of a natural object within a limited range of scales. However, the resulting task can be very difficult.

Thus, the physical value of the fractal approach is very limited. Furthermore, if the fractal scaling has a small range that spans only 1.5 or 2 orders of magnitude, then fractals do not provide a scale-independent description of surface roughness.

Power Spectral Density Function (PSDF) Approaches to Rough Surfaces

Currently, another trend is quite popular, namely the description of rough surfaces using exclusively the PSDF (power spectral density function) of the surface relief [28]. By Fourier transformation of expression (2) for R(δ), we obtain the power spectrum G(ω) or the power spectral density function (PSDF). If the frequency of the signal is denoted by ω, then the PSDF is defined as:

$$G(\omega) = \frac{2}{\pi}\int_0^\infty R(\delta)\cos\omega\delta\ d\delta. \tag{5}$$

Developing the random signal approach, Sayles and Thomas [29] presented experimental relationships between wavelength and scaled power spectral density for many different surfaces. They argued that the scaled spectral density functions of many surface profiles can be approximated as G(ω) = 2πΛ/ω2. Sayles and Thomas [29] called Λ the surface topothesis.

Borodich et. al. in [25] showed that models based solely on the power spectral density function (PSDF) are quite similar to fractal models, and these models do not reflect the tribological properties of surfaces. In particular, it is shown that different profiles can have the same PSDF.

Conclusions

An overview of mathematical approaches to the description of the topography of engineering surfaces is given. It is noted that, despite a fairly large number of parameters used to characterize the surface relief, only some parameters are quite useful. However, their use is quite limited, for example these parameters may be useful at the meso- or even micro-scale, but they may be useless at the nano-scale.

There are many models of random processes, but only the case of Gaussian processes is well developed. An analysis of the publications showed that undamaged surfaces are quite often Gaussian at both the micro- and nanoscales, while polished surfaces are not normal.

Based on the analysis of literary sources, an information model for the study of the structural characteristics of the engineering surface based on fractal analysis and the classification of experimental methods for determining the fractal dimension of statistically similar structures have been developed. Some shortcomings of fractal approaches and typical incorrect statements about fractals are identified. It is argued that the practical utility of fractal approaches is quite questionable. It should not be expected that the use of a mathematical fractal model of a rough surface will give significant advantages. Usually, such models are mathematically complex. Thus, a strict approach to fractal modeling can only replace a complex problem with another, more complex than the original one. In addition, the dimensions of physical (natural) fractals cannot be used as scale-independent parameters. Adequate explanations of the fractal concepts used must also be provided, otherwise results may be misinterpreted.

Surface roughness models based solely on the properties of the autocorrelation function or its Fourier transform (PDSF) are also discussed. It was pointed out that the PDSF approach to non-Gaussian surfaces does not reflect the tribological properties of the surfaces.

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Драч І.В., Диха М.О., Бабак О.П., Ковтун О.С. Моделювання поверхневої будови матеріалів триботехнічного призначення

Сучасна трибологія дає можливість правильно розраховувати, діагностувати, прогнозувати й підбирати відповідні матеріали пар тертя, призначати оптимальний режим роботи трибоз'єднання. Основним параметром для вирішення проблем тертя та інших проблем трибології є топографія поверхні. Основне призначення моделей в цих задачах – відображення трибологічних властивостей інженерних поверхонь. В рамках класичного підходу топографія поверхні досліджується на основі її зображень з точки зору функціональних і статистичних характеристик: оцінки функціональних характеристик мають за основу максимальну шорсткість за висотою і середню шорсткість по центральній лінії, а статистичні характеристики оцінюються за допомогою спектра потужності або функції автокореляції. Однак, ці характеристики не є лише властивостями поверхні. Вони залежать від роздільної здатності приладу для вимірювання геометрії поверхні та довжини сканування. Однак, ступінь складності форми поверхні можна подати через параметр, який називається фрактальною розмірністю: вищий ступінь складності має більше значення цього параметра. Фрактальна розмірність є характеристикою рельєфу поверхні та дає можливість пояснити трибологічні явища без впливу роздільної здатності. У цій статті подано огляд математичних підходів до опису рельєфу інженерних поверхонь, зокрема статистичне, стохастичне і топологічне моделювання, їх обмеження, переваги і недоліки. Обговорюється впровадження принципів теорії фрактальных структур, що дає можливість увести в аналіз структуроутворення в поверхневих і приповерхневих шарах матеріалів ступінь нерівноважності трибологічної системи й описати розвиток процесів тертя й зношування. Саме це є основою керування структурою поверхневих шарів матеріалів із заданими властивостями. Концепція фракталів, використовувана для кількісного опису дисипативної структури зони трибоз'єднання, дозволяє встановити зв'язок її фрактальної розмірності з механічними властивостями, а також критичними станами деформації металів і сплавів. Розглянуто хід дослідження і етапи фрактального моделювання, класифікацію методів фрактального аналізу структури інженерних поверхонь контакту. Подано критичний аналіз сучасних моделей, які мають за основу енергоспектральну функцію щільності, і є досить схожими на фрактальні моделі. Очікується, що читачі отримають огляд розвитку досліджень існуючих методів моделювання та напрямки майбутніх досліджень у галузі трибології.

Ключові слова: рельєф поверхні; статистичні моделі; стохастичні моделі; фрактальні моделі; енергоспектральна функція щільності